Unit 4: Circles and Volume

GSE Geometry

**Name: \_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_**

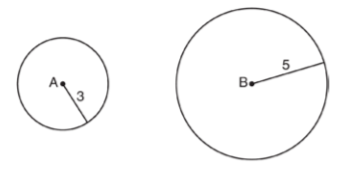
**Period: \_\_\_\_\_**

**2018 – 2019 Geometry**

**Pebblebrook High School**

**Mrs. Deuire**

Part A: 4.1 – 4.4

**4 – 1 Similar Circles and Proportions**

All circles are **similar** if there exists a **sequence of transformations** that maps circle A onto circle B.

In the diagram to the right, Circle A can be mapped onto circle B by first **rotating** circle A onto circle B, and then **dilating** circle A, centered at A, by a scale factor of .

Since there exists a sequence of transformations that maps circle A onto circle B, circle A is **similar** to circle B.

Performing a translation and dilation can transform a circle into any other circle. In other words, all circles are similar.

**Example 1**

Find the circumference of circle 1 and circle 2.

10

5

Write the ratio of circumference to the radius.

Determine whether the two circles are similar.

The length of the arc intercepted by a central angle is **proportional** to the **radius**.

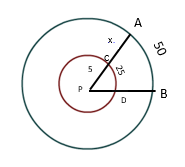
The diagram at the right shows two circles with the same center called **concentric circles**. It has already [been shown](https://mathbitsnotebook.com/Geometry/Circles/CRProveSimilar.html) that concentric circles are similar under a dilation transformation.

The scale factor is found by the following:

The same dilation that mapped the **smaller** circle onto the **larger** circle will also map the sector of the smaller circle with an **arc length** of s1 onto the slice sector of the larger circle with an arc length of s2. When the radius gets dilated by a scale factor, the arc length is also dilated by that same scale factor.

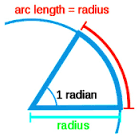
An equivalent proportion can be written as  . This proportion shows that the ratio of the arc length intercepted by a central angle to the radius of the circle will always yield the same (constant) ratio.

**Example 2:**

In the concentric circles below, = 50 and = 25

The length of PC is 5.

Find x.

**4 - 2 Circumference and Arc Length of a Circle**

Angles can be measured in units of either **\_\_degrees\_\_** or **\_\_radians\_\_**. A complete revolution is defined as **\_\_3600\_\_** or **\_\_\_\_**radians.

It is easy to use the fact that 360˚ = 2π radians to convert between the two measures.

When converting radians to degrees or degrees to radians, use the proportion

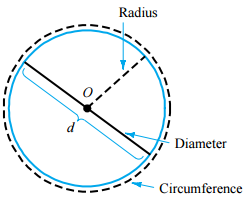


**Example 1**

Find the measure of each angle in radians.

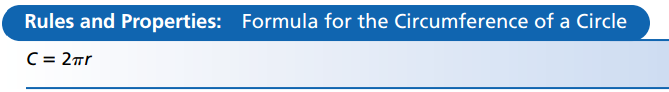


Find the measure of each angle in degrees.



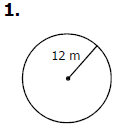
The distance around the outside of a circle is closely related to the concept of **\_\_perimeter\_\_**. We call the perimeter of a circle the **\_\_circumference\_\_**.

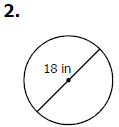
The distance across the circle through its center is the **\_diameter\_**. The **\_radius\_** is the distance from the center to a point on the circle. The diameter is always **\_double\_** the radius.

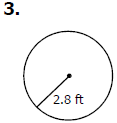
It was discovered long ago that the **\_ratio\_** of the circumference of a circle to its diameter always stay the same. The ratio has a special name, **\_\_\_\_**. Pi is approximately **\_\_\_\_** or **\_\_3.14\_\_**.

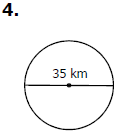
**Example 2**

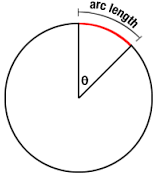
Find the circumference of each circle.









One way to measure arcs is in **\_\_degrees\_\_**. This is called the **\_\_angle measure\_\_** or degree measure. Arcs can also be measured in length, as a portion of the **\_\_circumference\_\_**.

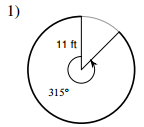
**\_\_Arc length\_\_** is the length of an arc or portion of a circumference. The arc length is directly related to the degree arc measure.

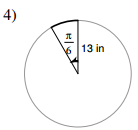
|  |  |
| --- | --- |
| **Central Angle in Degrees** | **Central Angles in Radians** |
| The formula for the arc measure is:  Where:  **C** is the **central angle** of the arc in degrees  **R** is the **radius** of the arc  **Π** is **Pi**, approximately 3.14 | If the central angle is in radians, the formula is simpler  Where:  **C** is the **central angle** of the arc in radians  **R** is the **radius** of the arc |

**Example 3**

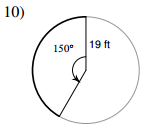
Find the arc length.

1.



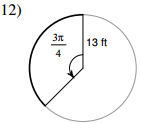


2.



3.

4.



**Example 4**

Find the missing variable. Round to nearest *hundredth* if necessary.

1. Arc Length = 66 2. Arc Length = 135

Central angle = Central Angle = 55

Radius = Radius =

3. Arc Length = 300 4. Arc Length = 28

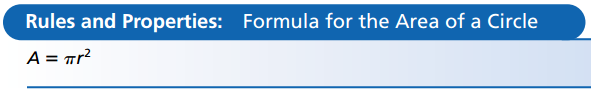
Central Angle = (in radians) Central Angle = (in degrees)

Radius = 6 Radius = 7

**4 – 3 Area and Area of a Sector**

The number pi (π), which is used to find the circumference, is also used in finding the **\_\_area\_\_** of a circle.

To find the area of a circle, all you need to know is its **\_\_radius\_\_**. We will leave our answers in terms of π, unless otherwise specified.



**Example 1**

Find the area.

1.



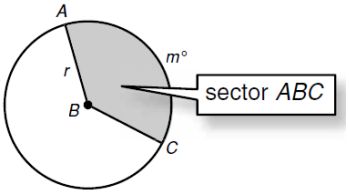
2.





4.

3.

An **\_\_area of a sector\_\_** is a region bounded by two radii of the circle and their intercepted arc.

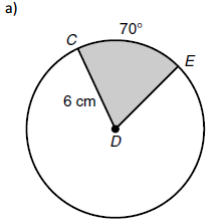
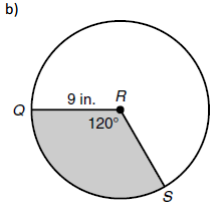
In degrees, to find the area of the sector you will multiply the **\_\_area\_\_** of the circle to the **\_\_angle ratio\_\_** of the circle.

|  |  |
| --- | --- |
| **Central Angle in Degrees** | **Central Angles in Radians** |
| The formula for the area of a sector is:  Where:  is the **central angle** of the arc in degrees  **R** is the **radius** of the arc  **Π** is **Pi**, approximately 3.14 | If the central angle is in radians, the formula is simpler  Where:  is the **central angle** of the arc in radians  **R** is the **radius** of the arc |

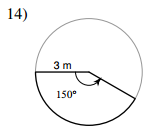
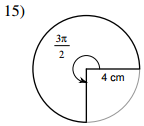
**Example 2**

Find the area of each sector.

**1.**



**2.**



4.

3.

**Example 3**

Find the missing variable.

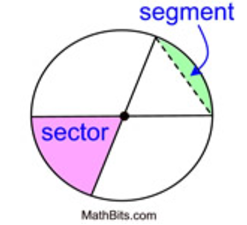
1. Area of Sector = 72 2. Area of Sector = 125

Radius = 4 Radius =

Central Angle = (in degrees) Central Angle = 10

**Area of a Segment of a Circle**

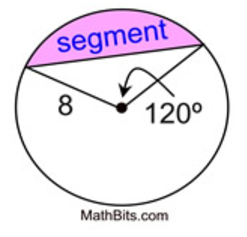
The segment of a circle is the region bounded by a chord and the arc subtended by the chord. The segment is the small partially curved figure left when the triangular portion of the sector is removed.



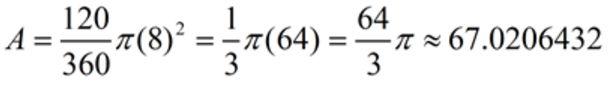
To find the area of the segment:



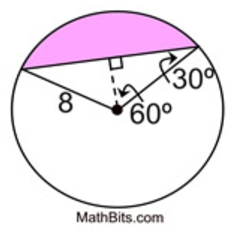
**Example 4:** Find the area of a segment of a circle with a central angle of 120 degrees and a radius of 8 cm. Express answer to the nearest integer.



**Step 1:** Find the area of the sector



**Step 2:** Draw the line that represents the height (altitude) of the triangle.



y

x

**Step 3:** Use the appropriate Trig ratio to find the missing sides

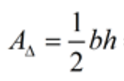
x = height of the triangle y = base of the triangle

Sin 60°= Cos 60°=

8 ·sin 60 = y 8 ·cos 60 = x

6.93 = y 4 = x

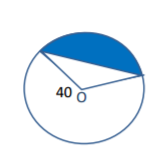
**Step 4:** Find the area of the triangle.



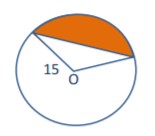
= = 27.7

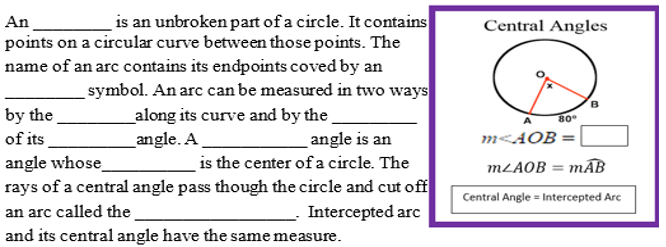


Asegment = 67.02 – 27.7 = 39.3 cm2

**Example 5**: Find the area of a segment of a circle if the central angle of the segment is 165º and the radius is 40.

**Example 6:** Find the area of a segment of a circle if the central angle of the segment is 50º and the radius is 15.



**4 – 4 Central and Inscribed Angles**

**arc**

**arc-like**

**measure**

**length**

**central**

**central**

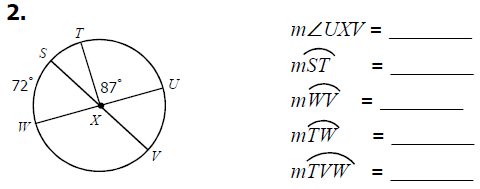
**vertex**

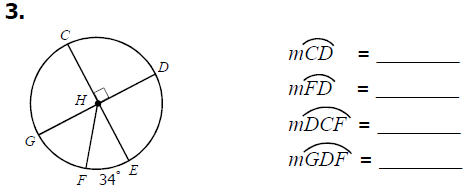
**intercepted arc**

**Example 1**

Find each angle and arc measures.

**1.**



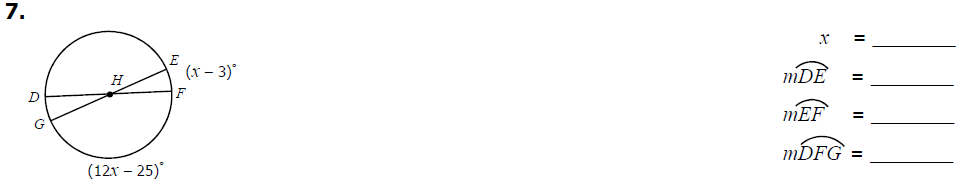


**2.**

**Example 2**

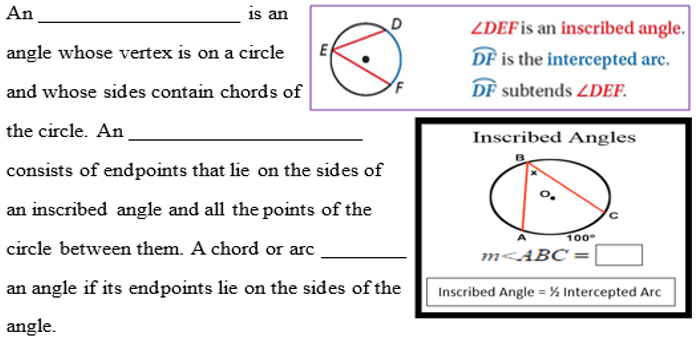
Find the measure of x and each arc measure.

**1.**





**2.**

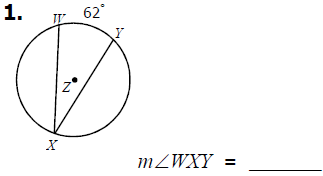


**inscribed angle**

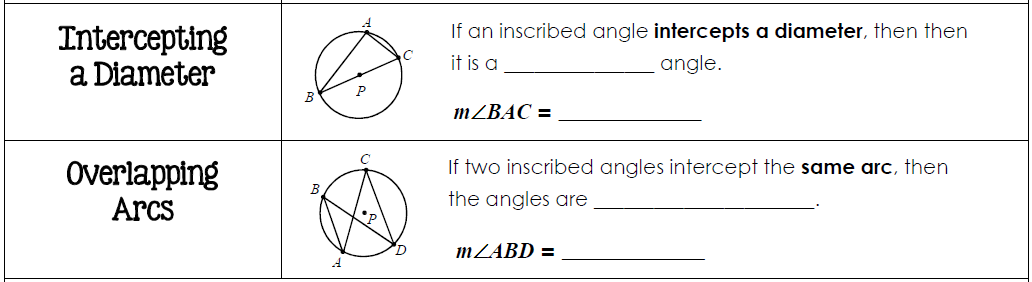
**intercepted arc**

**subtends**

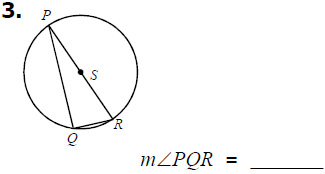
**Example 3**

Find each angle and arc measure.

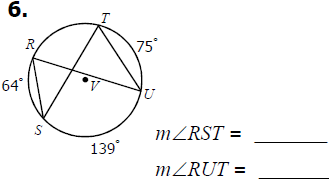




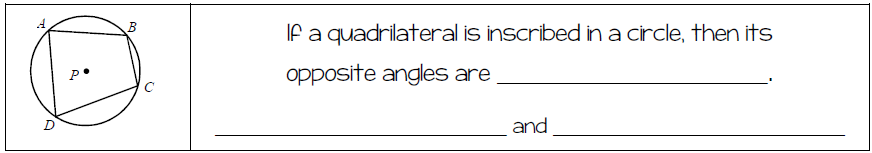
**Example 4**

Find each angle measure.

**1.**



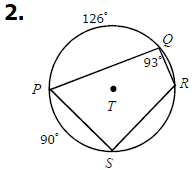
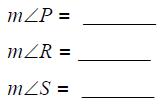
**2.**



**Example 5**

Find each value or measure.

**1.**



**2.**

